Modeling Growth:
Neoclassical and Schumpeterian growth models

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Co-organised by
Foundation for European Progressive Studies (FEPS)
Greenwich Political Economy Research Centre (GPERC)

University of Greenwich

This event received funding from the European Parliament

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Abstract

1. Revisit the theoretical origins of empirical growth models
2. Discuss Solow and augmented Solow models
3. Both models: No explicit distinction between capital accumulation and technological progress.
4. Augmented Solow model: treats human capital as additional input, but technology is still exogenous
5. Discuss Schumpeterian growth models with creative destruction and institutions
6. Schumpeterian models can address a wider range of questions – about policy, institutions, etc.
7. Innovations in growth modeling – incorporation of institional quality, product-market competition and non-linearities.

Plan

1. The Solow model with exogenous technology

2. Semi-endogenous augmented Solow model

3. Schumpeterian growth models
   1. Innovation – driven growth
   2. Democracy, innovation and growth

1. Conclusions

1. The Solow model with exogenous technology

Assumptions:

- No prices are involved - interested in real output as a measure of real income.
- No choice in terms work/leisure (all workers work) or savings (everybody saves a fixed portion of income).
- Savings are always invested.
- Output is shared between capital and labour in accordance with their marginal products.
- No government (and hence no taxes or subsidies)
- No international trade or financial markets.

1. The Solow model with exogenous technology

- Solow aims to address an essential problem in growth models without technology.
- Without technology per-capita output and per-capita capital do not grow at the steady-state.
- This is inconsistent with empirical evidence - most advanced economies exhibit growth in per-capita variables in the long run.
- Technology (*A) is added* into the model as follows:

$$Y(t) = f\,[K(t), A(t)L(t)] = K(t)^{\alpha}\,[A(t)L(t)]^{1-\alpha} \qquad (1)$$

- *Y* is real output, *K* is capital stock, *A* is technology, *L* is labour, *AL* is effective labour and α is elasticity of output with respect to capital stock.

- *AL* implies that labour is more productive when the level of technology is higher (i.e., technology is labour-augmenting or Harrod-neutral).

1. The Solow model with exogenous technology

- **Technological progress** and **population growth** are **exogenous**.

- Hence, current levels of technology, labour and effective labour at year (t) can be expressed as functions of initial values:

$$A(t) = A(0)e^{gt};$$

$$L(t) = L(0)e^{nt} \quad \text{and}$$

$$[A(t)L(t)] = [A(0)L(0)]e^{(n+g)t}$$

Where A(0), L(0) and [A(0)L(0)] are initial levels; and g, n and $n+g$ are growth rates technology, labour and effective labour.

1. The Solow model with exogenous technology

- Define the following ratios:

$$s = \frac{S}{Y} \qquad Saving\ rate;$$

$$k = \frac{K}{AL} \qquad Capital\ stock\ per\ effective\ labour$$

$$y = \frac{Y}{AL} \qquad Output\ per\ effective\ labour$$

- Then (1) can be written as:

$$y(t) = \frac{K(t)^{\alpha}}{AL(t)^{\alpha}} = k(t)^{\alpha} \qquad (2)$$

- According to (2), output per effective labour in year (t) is a positive function of capital stock per effective labour k(t) in that year.

1. The Solow model with exogenous technology

Define the evolution of *k(t):*

$$\dot{k}(t) = sy(t) - (n + g + \delta)k(t) = sk(t)^{\alpha} - (n + g + \delta)k(t)$$

- Steady-state occurs when $\dot{k}(t) = 0$. This yields a steady-state value of capital-to-effective-labour ratio (k*) in (3).

$$sk^{*\alpha} - (n + g + \delta)k^{*} = 0$$

$$\rightarrow\ sk^{*\alpha} = (n + g + \delta)k^{*}$$

$$\rightarrow\ k^{*} = [s/(n + g + \delta)]^{1/(1-\alpha)} \qquad (3)$$

1. The Solow model with exogenous technology

Now substitute the steady-state value of capital - K* = k*[A(t)L(t)] = $[s/(n+g+\delta)]^{1/(1-\alpha)}$[A(t)L(t)] - into the Cobb-Douglas production function in (1).

$$Y(t) = \left[s/(n+g+\delta)^{1/(1-\alpha)}A(t)L(t)\right]^{\alpha}[A(t)L(t)]^{(1-\alpha)}$$ or

$$Y(t) = \left[s/(n+g+\delta)^{\alpha/(1-\alpha)}\right][A(t)L(t)] \qquad (4)$$

Take logs of both sides of (4):

$$lnY(t) = \frac{\alpha}{1-\alpha}lns - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + lnA(t) + lnL(t) \qquad (5)$$

1. The Solow model with exogenous technology

Express (5) in terms of output per worker:

$$lnY(t) - lnL(t) = \frac{\alpha}{1-\alpha} lns - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + lnA(t) \quad (6)$$

Recall that $A(t) = A(0)e^{gt}$, then:

$$lnY(t) - lnL(t) = \frac{\alpha}{1-\alpha} lns - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + lnA(0) + gt$$

Let $lnA(0) = \theta + \varepsilon$, then:

$$lnY(t) - lnL(t) = lny(t) = \theta + gt + \frac{\alpha}{1-\alpha} lns - \frac{\alpha}{1-\alpha} \ln(n+g+\delta) + \varepsilon$$

(7)

1. The Solow model with exogenous technology

Equation (7) is in levels – but can be converted into a growth equation by taking the log difference between income in year t and income T years ago, giving:

$$\ln y_t - \ln y_{t-T} = \Delta y_t = \theta + gt - \gamma_0 \ln y_{t-T} + \frac{\alpha}{1-\alpha} lns - \alpha/(1-\alpha)\ ln(n+g+\delta) + v \qquad (8)$$

In (8), the convergence rate is γ_0/T.

Assuming that the error term (v) is not correlated with the regressors *s*, *n*, *g* and δ; equation (8) can be estimated with:

- OLS if data is averaged over the whole period. In this case, the term *gt* disappears.

  OR
- Dynamic panel data methods (e.g., GMM) if data has a panel structure, with averaging over shorter time periods. In this case, *gt* is captured by period time dummies

2. Augmented Solow model

- Omission of human capital in the original Solow model is problematic from theoretical and empirical perspectives.
- Kendrick (1976) argued that more than 50% of the US capital stock in 1969 was human capital.
- Lucas (1988) argued that there may be decreasing returns to physical capital accumulation, but increasing returns to human capital accumulation.
- Hence, returns to total capital (physical + human capital) may be constant.
- Then, absence of human capital in (7) or (8) causes omitted variable bias (OVB).

2. Augmented Solow model

Then, augmented Solow model (with human capital) can be written as follows:

$$Y(t) = f\,[K(t), A(t)L(t)] = K(t)^{\alpha} H(t)^{\beta}\, [A(t)L(t)]^{1-\alpha-\beta} \qquad (9)$$

Here, capital is separated into physical capital (K) and human capital (H).

Following the routine above, we can write the augmented model as follows:

$$\ln y_t - \ln y_{t-T} = \Delta y_t = \theta + gt - \beta_0 \ln y_{t-T} - \left(\frac{\alpha+\beta}{1-\alpha-\beta}\right) \ln(n+g+\delta) + \left(\frac{\alpha}{1-\alpha-\beta}\right) \ln s_k + \left(\frac{\beta}{1-\alpha-\beta}\right) \ln s_h + v \qquad (10)$$

With convergence rate = β_0 / T

Criticisms of Neoclassical models

From a **heterodox perspective**, they are criticised for:

- Assuming full employment of labour and full capacity utilization
- Assuming no effective demand failures
- Having no ‘investment function’ in addition to and independently of the savings function (they overlook the scope for investment-led growth)
- Overlooking reverse causality between output growth and labour productivity (Verdoorn’s Law)

From a **Schumpeterian perspective** (Aghion and Hovitt, 2006), they are criticised for:

- Failing to explain why the US has been growing faster than Europe since the mid-1990s - even though the average European saving rate has been higher than the US rate. Also, the average European capital-labour ratio has remained higher than the US ratio and has not decreased.
- Failing to explain why the growth gap between Europe and the US has persisted despite the fact that the institutions of property rights (which affect technology adoption) have been similar.

3. Schumpeterian models

In Schumpeterian theory aggregate output is produced by a continuum of intermediate products in accordance with:

$$Y = L^{1-\alpha} \int_0^1 A(i)^{1-\alpha} x(i)^{\alpha} di, \qquad (11)$$

In (11), product variety is normalized to unity and each intermediate product (xi) has a separate productivity parameter A(i).

Each sector is monopolized and produces its intermediate product with a constant marginal cost of unity. The monopolist in sector i faces a demand curve given by the marginal product:

$$\alpha . (A(i)L/x(i)^{1-\alpha} \qquad (12)$$

3.1 Schumpeterian models: Innovation-driven growth

Equating marginal revenue (α times the marginal product in 12) to marginal cost of unity yields the monopolist's profit-maximizing intermediate output:

$$x(i) = \varphi L A(i), \quad \text{where } \varphi = \alpha^{2/(1-\alpha)} \tag{13}$$

Using this to substitute for each x(i) in the production function (11) yields the aggregate production function:

$$Y = \theta A L \tag{14}$$

where $\theta = \varphi^{\alpha}$ and A is the average productivity parameter: $A \equiv \int_0^1 A(i)\, di$

Innovations in Schumpeterian theory create improved versions of old products. An innovation in sector i consists of a new version whose productivity parameter A(i) exceeds that of the previous version by the fixed factor $\gamma > 1$. (We can call γ as the productivity premium on innovation)

3.1 Schumpeterian models: Innovation-driven growth

Suppose that the probability of an innovation arriving in sector i over any short interval of length dt is $\mu.dt$.

Then the growth rate of A(i) is

$$\frac{d(A(i))}{A(i)}.\frac{1}{dt} = \begin{cases} (\gamma - 1).dt \; with \; probability \; of \; \mu.dt \\ 0 \qquad with \; probability \; of \; 1 - \mu.dt \end{cases} \qquad (15)$$

Here, μ is the flow probability of innovation.

Therefore the expected growth rate of A(i) is:

$$E(g) = \mu(\gamma - 1) \qquad (16)$$

In any sector, the probability of innovation (μ) is a function of R&D expenditures and productivity:

$$\mu = \lambda R/A \qquad (17)$$

3.1 Schumpeterian models: Innovation-driven growth

In (17), R is the amount of final output spent on R&D; and λ is a flow parameter

Dividing R by the productivity parameter (A) takes into account the force of increasing complexity.

That is, as productivity increases the society must undertake more R&D expenditures just to keep innovating at the same rate as before.

The law of large numbers guarantees that the growth rate g equals the expected growth rate in (16). Thus, from (16) and (17) we have:

$$g = (\gamma - 1)\lambda R/A \qquad (18)$$

Let's define the fraction of GDP spent on R&D as:

$$n = R/Y \qquad (19)$$

Combining (14), (18) and (19), we obtain:

$$g = (\gamma - 1)\lambda\theta n L \qquad (20)$$

Thus, Schumpeterian models imply that the way to grow rapidly is:

- Not to save a large fraction of output; but
- To devote a large fraction of output (n) to R&D.

Further implications of Schumpeterian models:

- The higher the productivity premium (γ) the faster is growth
- Reduced market power (increased competition) reduces incentive for R&D and harms growth!
- But more recent models show that the relationship between competition and growth has an inverted-U shape

3.2 Schumpeterian models: democracy and growth

New Schumpeterian models draw on insights from institutional economics.

Acemoglu, Aghion and Zilibotti (2006) propose a model that differentiates between economic growth in developed and developing countries.

Growth in developing countries is driven by adoption and imitation of existing technologies and investment in existing lines of business.

Growth in advanced (frontier) economies is driven by innovation.

Other insights from AAZ (2006):

Equilibrium organization of production and the broader institutions of the society may differ depending on:

(a) the level of development; and

(b) the distance of the country's technology to the frontier technology.

3.2 Schumpeterian models: democracy and growth

c) Openness is more important for growth in close-to-frontier countries compared to distant-to-frontier countries

d) High entry barriers are more detrimental to growth as the country approaches the frontier

e) The more frontier an economy is, the more growth in this economy relies on research-oriented education

f) The correlation between democracy and innovation/growth is more positive and significant in more frontier economies.

In what follows, we will provide a proof of prediction (f).

In equilibrium, the innovation efforts is given by:

$$z_{jt} = \bar{z} = \frac{\beta\pi}{\lambda} \tag{21}$$

Here, $\bar{z}$ is equilibrium innovation effort; β is the level of democracy; π is profits and λ is cost of R&D flows.

3.2 Schumpeterian models: democracy and growth

Taking derivative with respect to democracy:

$$\frac{\delta \bar{z}}{\delta \beta} = \frac{\pi}{\lambda} > 0$$

Equilibrium innovation effort is increasing in democracy level:

Democracy fosters higher levels of innovation as it reduces barriers to entry.

Now we can turn to the relationship between democracy and growth.

The average productivity of a country at the **beginning** of the period

$$A_{t-1} = \int_0^1 A_{jt} d_j = \mu \bar{A}_{t-1} + (1-\mu)\bar{A}_{t-2} \qquad (21)$$

3.2 Schumpeterian models: democracy and growth

Average productivity at the end of the period is:

$$A_t = \mu[\beta\bar{z}\gamma\bar{A}_{t-1} + (1-\beta\bar{z})\bar{A}_{t-1}] + (1-\mu)\bar{A}_{t-1} \qquad (22)$$

In (21) amd (22), $\bar{A}$ is productivity; μ is probability of innovation; β is the level of democracy experienced; and $\bar{z}$ is innovation effort.

Then the growth rate of average productivity over the period is:

$$g_t = \frac{A_t - A_{t-1}}{A_{t-1}} = \gamma\frac{\mu\beta\bar{z}(\gamma-1)+1}{\mu(\gamma-1)+1} \qquad (23)$$

3.2 Schumpeterian models: democracy and growth

Taking partial derivative with respect to democracy (β), we can see that democracy is growth enhancing:

$$\frac{\delta g_t}{\delta \beta} = (\bar{z} + \frac{\delta \bar{z}}{\delta \beta}\beta)(\frac{\gamma\mu(\gamma-1)}{\mu(\gamma-1)+1} > 0 \qquad (34)$$

Moreover, democracy is more growth enhancing the closer the country is to the world technology frontier (i.e., the higher is the probability of innovation):

$$\frac{\delta^2 g_t}{\delta \beta \delta \mu} = (\bar{z} + \frac{\delta \bar{z}}{\delta \beta}\beta)(\frac{\gamma(\gamma-1)}{[\mu(\gamma-1)+1]^2} > 0 \qquad (35)$$

Conclusions

- Despite restrictive assumptions, Neoclassical models have been successful in estimating factor shares until mid-1990s.
- However, they are silent on skill-biased technical change and falling labour share
- Convergence rates estimated with Neoclassical models vary between studies - with a mean of 4.3% and minimum and maximum values of 1.43% and 8.34% respectively (Abreu et al., 2005). These rates are much higher than the 2% implied factor shares of 79% and 30% for labour and capital respectively;
- Neoclassical models have also been criticised for failing to explain the difference in growth rates of Europe and the US despite similar institutional characteristics and similar or even higher saving rates or capital-labour ratio in Europe.

Conclusions

- In Schumpeterian models, growth results from quality-improving innovations.
- Unlike neoclassical models, they highlight the importance of key economic variables such as the country's distance to the technological frontier, its institutional quality or its degree of openness.
- This feature enables Schumpeterian models to address policy-relevant questions.
- However, Schumpeterian models share the same weakness as neoclassical models: they overlook the effects of deficient demand or other demand-side constraints stemming from balance of payments deficits.